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COMMENTS ON A PAPER BY W. ROTHFARB, N. P. SCHEIN, AND I. T. FRISCH: "COMMON TERMINAL MULTICOMMODITY FLOW"

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The article "Common Terminal Multicommodity Flow," by W. Rothfarb, N. P. Schein, and I. T. Frisch, which appeared as a letter to the editor in the January-February 1968 edition of Operations Research, describes an algorithm that solves a specific class of common terminal multicommodity network flow problems. This note shows that this class of problems can also be formulated as single commodity minimal cost flow problems for which a number of solution techniques have been developed.

The network they treat consists of a finite set of nodes, n of which are sources and one of which is a common terminal. Pairs of nodes are joined by directed arcs, each of which has a non-negative capacity. The flow of any commodity must be conserved at all nodes other than its source and the common terminal and total flow on any arc must not exceed its capacity. The problem they treat on this network is that of finding a flow pattern which maximizes $\sum_{i=1}^{n} \alpha_i f_i$ where f_i denotes the amount of flow of commodity i from the i^{th} source node, i, to the terminal node, t, and $\alpha_1 > \alpha_2 > \ldots > \alpha_n \geq 0$. Henceforth, this will be referred to as problem 1.

A problem which their's is equivalent to is the following one. Let the network consist of a single source, s, instead of the n sources, 1, ..., n. Furthermore, let the arcs have costs in addition to capacities. The flow of the single commodity must be conserved at all nodes except s and t and as in problem 1 the flow on any arc cannot exceed its capacity. It is required to find a flow pattern that minimizes $\sum c_{jk} x_{jk}$ where c_{jk} is the cost and x_{jk} the flow on (j,k), the arc directed from j to k. This will be referred to as problem 2.

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The conversion of problem 1 to problem 2 proceeds as follows. To the problem 1 network, add one node, s, and n arcs, (s,1), (s,2), ..., (s,n). Let $c_{si} = -\alpha_i$ for i = 1, ..., n and set all other $c_{jk} = 0$. Let the capacity of the new arcs be infinite (or, if finiteness is required, at least as great as the sum of the capacities of the original arcs). For any feasible flow pattern on the newly defined problem 2 network, the flows on arcs not joined to s constitute a feasible flow for the problem 1 network with $\sum_{i=1}^{n} \alpha_i f_i = -\sum_{j,k} c_{jk} x_{jk}$. The reverse also follows. Namely, a feasible flow pattern for the problem 1 network specifies a feasible flow pattern for the new network with $x_{si} = f_i$, i = 1, ..., n and $\sum_{j,k} c_{jk} x_{jk} = -\sum_{i=1}^{n} \alpha_i f_i$. Thus these two network problems are equivalent and problem 1 is a special case of problem 2.

A number of algorithms for solving problem 2 have been developed, some of which may be found in references 1, 2, 5, and 6. While several of these references specify that arc costs be non-negative, each presents algorithms which are valid if non-negativity is merely required for the total cost on all directed cycles. All directed cycles on networks constructed from problem 1 are zero cost. This follows from the fact that all arcs with non-zero cost are joined to s and no arc terminates at s (i.e. no arcs of the form (k,s)).

Alternatively, problem 1 may be converted to one of finding a minimum cost circulation flow. This differs from problem 2 only in that flow must be conserved at all nodes, including s and t. Such a formulation is obtained from problem 2 by adding the arc (t,s) and assigning it zero cost and infinite capacity. Flow on (t,s) would be equal to the total flow from s (and consequently the total flow from 1,2, ..., n) to t. A method for solving this is found in reference 4.

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